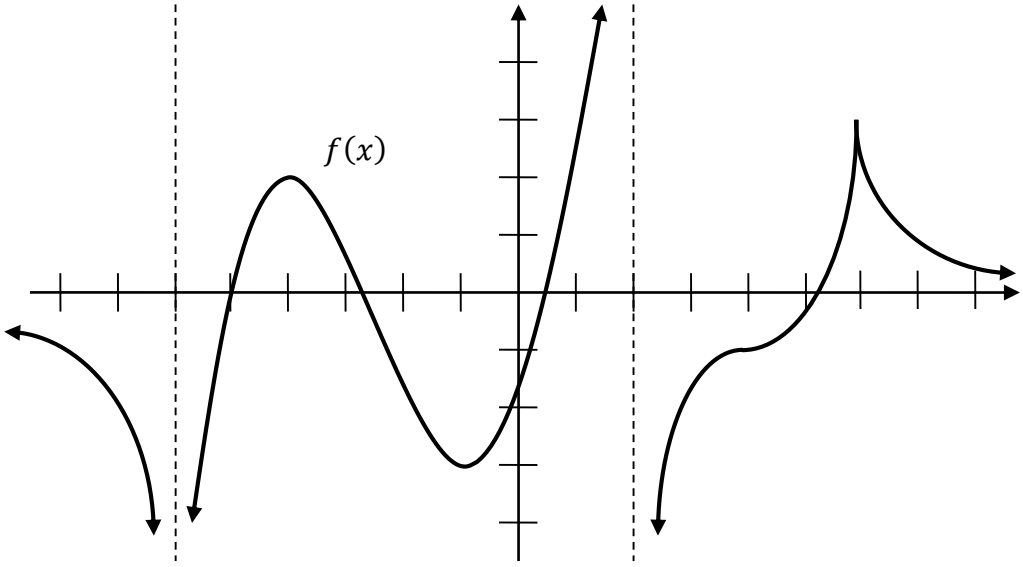
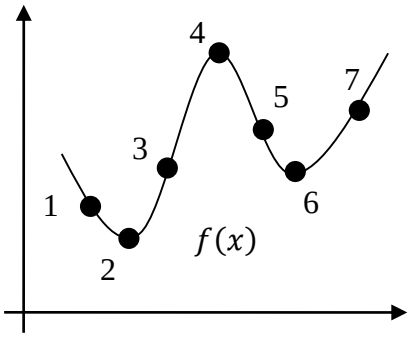
Worksheet 10 – Increasing/Decreasing, Concavity, and Curve Sketching

1. Use the graph of $f(x)$ below to answer parts (a) through (h).



(a) Find the intervals where $f(x)$ is increasing.
(b) Find the intervals where $f(x)$ is decreasing.
(c) Find all the *x*-values where the slope of $f(x)$ is zero.
(d) Find all the *x*-values where the derivative of $f(x)$ does not exist.
(e) Find all the critical points of $f(x)$.
(f) Find the coordinates of all the relative maxima of $f(x)$.
(g) Find the coordinates of all the relative minima of $f(x)$.
(h) Find all the *x*-values where $f(x)$ changes from increasing to decreasing or decreasing to increasing.

2. Use the numbered points on the graph of $f(x)$ at the right to answer parts (a) through (h).



(a) At which points is the function increasing?
(b) At which points is there a relative maximum?
(c) At which points is the slope negative?
(d) At which points is the slope zero?
(e) At which points is there a relative minimum?
(f) At which points is the function decreasing?
(g) At which points is the slope positive?

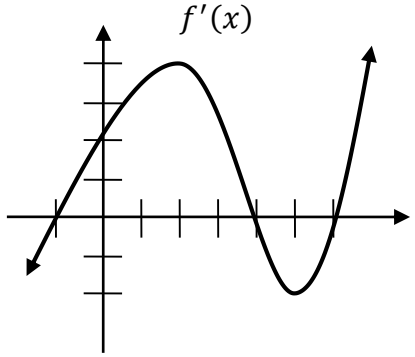
3. Sketch the graph of a single continuous function with a *y*-intercept at $(0,2)$ that has the following properties.

(1) $f'(x) < 0$ on $(-\infty,-6) \cup (1,3)$
(2) $f'(x) > 0$ on $(-6,1) \cup (3,\infty)$
(3) $f''(x) > 0$ on $(-\infty,-6) \cup (3,\infty)$
(4) $f''(x) < 0$ on $(-6,3)$

4. Use the second derivative test to find the local extrema of $f(x) = x^3 - 3x$.

5. Use the graph of $f'(x)$ at the right to answer parts (a) through (i).

(a) What is the slope of $f(x)$ at $x = 2$?
(b) For which *x*-values does $f(x)$ have a horizontal tangent line?
(c) Find the intervals where $f(x)$ is increasing.
(d) Find the *x*-values where $f(x)$ has a relative minimum.
(e) Find the *x*-values where $f(x)$ has a relative maximum.
(f) Is $f(x)$ increasing or decreasing at $x = 5$?
(g) Is $f(x)$ concave up or concave down at $x = 4$?
(h) Find the *x*-values where $f(x)$ has an inflection point.
(i) Find $\lim_{x\to-\infty} f(x)$.



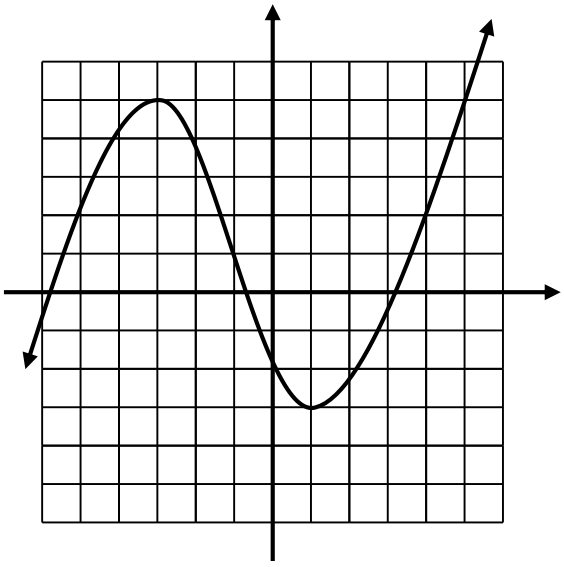
6. Match each observation 1-5 to each conclusion A-E.

(1) The point $(3,4)$ is on the graph of $f'(x)$.
(2) The point $(3,4)$ is on the graph of $f(x)$.
(3) The point $(3,4)$ is on the graph of $f''(x)$.
(4) The point $(3,0)$ is on the graph of $f'(x)$ and $(3,4)$ is on the graph of $f''(x)$.
(5) The point $(3,0)$ is on the graph of $f'(x)$ and $(3,-4)$ is on the graph of $f''(x)$.

(A) $f(x)$ has a relative minimum at $x = 3$.
(B) At $x = 3$, the graph of $f(x)$ is concave up.
(C) At $x = 3$, the tangent line to the graph of $f(x)$ has slope 4.
(D) At $x = 3$, the value of $f(x)$ is 4.
(E) $f(x)$ has a relative maximum at $x = 3$.

7. In the graph of $f(x)$ at the right, assume that $(-1,1)$ is an inflection point. Use the graph to answer parts (a) through (k).

(a) Find the intervals where $f(x)$ is decreasing.
(b) Find the intervals where $f'(x) > 0$.
(c) Find all the critical points.
(d) Find the intervals where $f(x)$ is concave up.
(e) Find the intervals where $f(x)$ is concave down.
(f) Is $f'(-4)$ positive, negative or zero?
(g) Is $f''(-4)$ positive, negative, or zero?
(h) Is $f'(0)$ positive, negative or zero?
(i) Is $f''(0)$ positive, negative or zero?
(j) Is $f'(1)$ positive, negative, or zero?
(k) Is $f''(1)$ positive, negative, or zero?



8. Await the instructor directions for the following functions.

(a) $f(x) = x^3 + 3x^2 - 9x + 4$
(b) $f(x) = 1 + x^{2/3}$
(c) $f(x) = \frac{x^2+4}{2x}$
(d) $f(x) = \arcsin(1/x)$
(e) $f(x) = x^4 - 4x^3 + 10$
(f) $f(x) = 6x^{2/3} - 4x$
(g) $f(x) = \frac{(x+1)^2}{1+x^2}$
(h) $f(x) = e^{2/x}$